given airplane flying at a given C_L can substantially reduce its trailing vortex upset potential by deploying its flaps and altering its flight attitude while maintaining its C_L. This concept is consistent with the theoretical prediction by Rossow[8] which indicates that additional unloading of the outer portion of the wing (relative to the inner portion) by the use of inboard flaps will produce further reductions in the maximum vortex wake velocities. These ideas and findings might be taken into consideration along with performance and noise considerations in the selection of aircraft approach L/Ds.

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Explicit Model Following Control Scheme Incorporating Integral Feedback

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Introduction

THE type one multivariable servomechanism problem has only recently been formulated and studied in detail. Porter and Powers, in a series of letters published in Electronics Letters, and Porter[1] have described various aspects of multivariable systems which incorporate integral feedback (IFB) as a part of the control law. Young and Willems[2] reformulate the problem in what they term as a precise manner and develop "explicit and general controllability conditions for such systems." They also develop for the type one servomechanism problem a pole assignment algorithm and an algorithm based on the use of model in the performance index (MPI) optimal control theory techniques.

This Note describes the extension of the previous work to explicit model following systems for the case where IFB is incorporated into both the model and the plant dynamics. In this formulation optimal feedback control gains for the model with integral feedback are first calculated. Then the explicit model following problem, with integral feedback in the plant, is posed and solved through the use of optimal control theory.[3] (This form of model following has been called a passive adaptive scheme.[4]) As a result of this formulation of the problem, the properties of type one servomechanism systems are embodied in both the model and plant response. These and other properties of the present formulation of the problem will be discussed later. Another important feature which should be noted in the sequel is that the overall system becomes a command augmentation system.

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Problem Formulation

Model

$$\dot{x}_m = A_m x_m + B_m u_m + d_m \qquad (1)$$

$$y_m = C_m x_m$$

Plant

$$\dot{x}_p = A_p x_p + B_p u_p + d_p \qquad (2)$$

$$y_p = C_p x_p$$

x, u, y are n, r, and q vectors, respectively. A, B, and C are compatible matrices. Furthermore, C is of a special form; it is a qxn output matrix consisting of zero row elements except for one unity element in each row. Thus, C is a "selection matrix" which selects those elements of x which are to incorporate IFB and which, as will become evident, are those variables for which command control is desired. d_m and d_p are n element model and plant constant bias vectors, respectively.

Integral feedback (IFB) may be incorporated (using model Eqs. (1) to illustrate this) by defining

$$\dot{z}_m \triangleq y_r - y_m = y_r - C_m x_m \qquad (3)$$

and augmenting the $\dot{x}_m$ equation with this equation. y_r is an external reference or input signal. Then defining

$$\hat{x}_m^T \triangleq [\, x_m^T \mid z_m^T \,]$$

The open-loop model dynamics are

$$\dot{\hat{x}}_m = \begin{bmatrix} A_m & O \\ -C_m & O \end{bmatrix} \hat{x}_m + \begin{bmatrix} B_m \\ O \end{bmatrix} u_m + \begin{bmatrix} d_m \\ y_r \end{bmatrix} \qquad (4)$$

Infinite time optimal linear regulator theory, with performance index

$$PI_m = \int_0^\infty (\|\hat{x}_m\|_{Q_1} + \|u_m\|_{R_1})\, dl$$

may be applied to obtain the gain matrices in the control law

$$u_m = -K_{PD_m} x_m - K_{I_m} z_m \qquad (5)$$

Q_1 and R_1 are chosen such that "good" transient response, such as that required by satisfactory aircraft handling qualities specifications, is realized for the closed-loop model dynamics

$$\dot{\hat{x}}_m = \begin{bmatrix} A_m - B_m K_{PDm} & -B_m K_{Im} \\ -C_m & O \end{bmatrix} \hat{x}_m + \begin{bmatrix} d_m \\ y_r \end{bmatrix} \qquad (6)$$

References 2 and 3 prove that the above is possible provided that (A,B) is a controllable pair and that the rank of $CA^{-1}B$ is q.

Figure 1a shows the block diagram for the closed-loop model with IFB which has the properties of 1) zero steady-steady error, 2) steady-state decoupling, 3) "good" transient response, and 4) immunity to constant bias error, d_m.

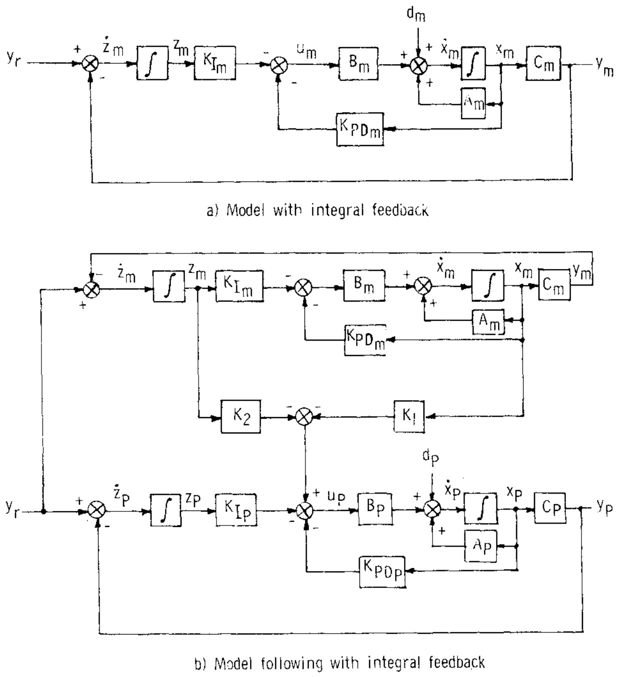


Fig. 1 Type-one servomechanism systems.

The open-loop plant, augmented with IFB in the same manner as the model, and with similar definitions becomes

$$\dot{\hat{x}}_p = \begin{bmatrix} A_p & O \\ -C_p & O \end{bmatrix}\hat{x}_p + \begin{bmatrix} B_p \\ O \end{bmatrix}u_p + \begin{bmatrix} d_p \\ y_r \end{bmatrix} \tag{7}$$

In explicit model following systems designed through the use of optimal linear regulator theory, the plant is forced to follow the model by augmenting the plant open-loop dynamics with the model closed-loop dynamics, forming a performance index with a term representing the error between model and plant states and a term representing the control required to force the plant to follow the model, and solving for the optimal gains. Defining $\tilde{x}^T = [\hat{x}^T \vdots \hat{x}_p{}^T]$, the pertinent equations are

$$\dot{\tilde{x}} = \begin{bmatrix} \begin{matrix} A_m - B_mK_{PD_m} & -B_mK_{Im} \\ -C_m & \end{matrix} & O \\ O & \begin{matrix} A_p & O \\ -C_p & O \end{matrix} \end{bmatrix}\tilde{x} + \begin{bmatrix} O \\ \hline B_p \\ O \end{bmatrix}u_p + \begin{bmatrix} d_m \\ y_r \\ d_p \\ y_r \end{bmatrix} \tag{8}$$

$$PI = \int_0^{\infty}(\|\hat{x}_m - \hat{x}_p\|_Q + \|u_p\|_R)dt \tag{9}$$

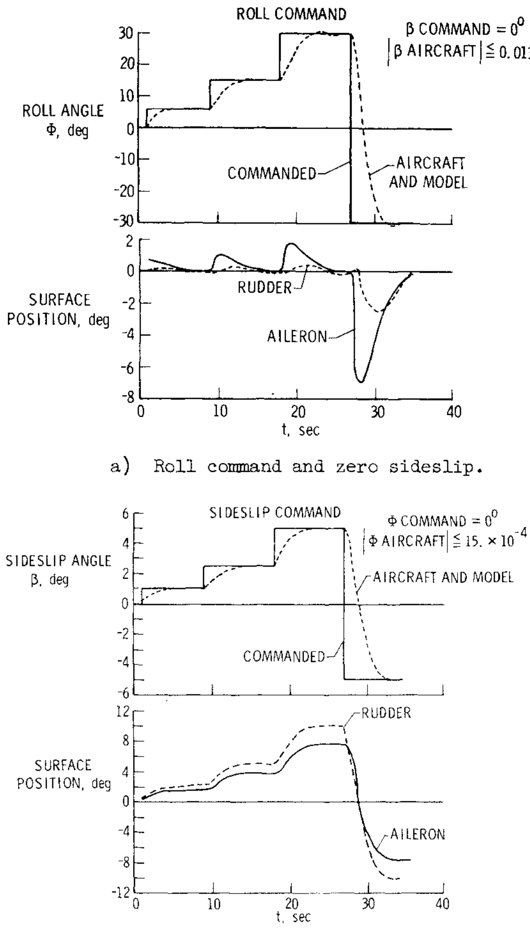
$$u_p = -K\tilde{x} \triangleq [K_1 \mid K_2 \mid K_{PD_p} \mid K_{I_p}]\tilde{x}$$

A block diagram illustrating the system is shown in Fig. 1b.

Note in Fig. 1b that the model and the plant diagram are the same except for the plant input from the model and the absence of a bias signal for the model whereas one is shown for the plant. No bias signal is shown for the model because all model computations would be done inside the computer and no bias signal would exist.

The plant response to the input signal y_r has the same advantages as previously listed for the model. However, "good" transient response for the plant is dictated by its model following characteristic. The quality of the model following characteristic is a function of the weighting matrices Q and R in the performance index and must be determined through numerical experimentation. A conflicting requirement of "keeping the gains low," especially those in the gain matrix K_{PD}, is also involved.

As a result of adding additional feedback loops to both the model and the plant and the fact that stability for the entire system is assured, reduced sensitivity to variations in the plant parameters and proportional-differential gains is also conjectured, but has not yet been proven.



a) Roll command and zero sideslip.

b) Sideslip command and zero roll.

Fig. 2 Typical time history results for simulated aircraft lateral dynamics.

Results

Computer stimulation results have been obtained for both lateral and longitudinal aircraft dynamics; however, data are presented for the lateral mode only. The lateral equations of motion of the aircraft studied, at a particular flight condition, are

$$\begin{bmatrix} \dot{p}_p \\ \dot{r}_p \\ \dot{\beta}_p \\ \dot{\phi}_p \end{bmatrix} = \begin{bmatrix} -3.6 & 0.197 & -35.2 & 0 \\ -0.0377 & -0.357 & 5.88 & 0 \\ 0.0688 & -0.996 & -0.216 & 0.073 \\ 0.9947 & 0.1027 & 0 & 0 \end{bmatrix}\begin{bmatrix} p_p \\ r_p \\ \beta_p \\ \phi_p \end{bmatrix} + \begin{bmatrix} 14.65 & 6.54 \\ 0.217 & -3.086 \\ -0.0054 & 0.05 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \delta_{a_p} \\ \delta_{r_p} \end{bmatrix} \tag{11}$$

where p, r, β, ϕ are roll and yaw rate and sideslip and bank angle, respectively, and δ_a and δ_r are aileron and rudder control surfaces, respectively.

In this example it will be assumed that command control over roll angle, ϕ, and sideslip angle, β, is desired. Then for both the model and aircraft

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{12}$$

The open-loop model dynamics used were

$$\begin{bmatrix} \dot{p}_m \\ \dot{r}_m \\ \dot{\beta}_m \\ \dot{\phi}_m \end{bmatrix} = \begin{bmatrix} -4. & 0.865 & -10. & 0 \\ -0.040 & -0.507 & 5.870 & 0 \\ 0 & -1.0 & -0.743 & 0.734 \\ 1.0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} p_m \\ r_m \\ \beta_m \\ \phi_m \end{bmatrix} + \begin{bmatrix} 20.0 & 3.30 \\ 0 & -3.13 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \delta_{a_m} \\ \delta_{r_m} \end{bmatrix} \tag{13}$$

Incorporating IFB and choosing

$$Q_1 = \text{diam}[0,0,0,0,1,1]; \; R_1 = \text{diam}[1,1]$$

provides model proportional differential and integral feedback gains

$$K_{PD_m} = \begin{bmatrix} -3.025 & -0.466 & 0.634 & -5.475 \\ -0.426 & 3.19 & -2.102 & -0.791 \end{bmatrix} K_{I_m} = \begin{bmatrix} -3.143 & 0.347 \\ -3.347 & -3.143 \end{bmatrix}$$

Next, IFB is incorporated into the aircraft dynamics. Various values for Q and R in the PI, Eq. (9), were investigated. The values

$$Q = \text{diam}[1, 1, 1, 1.\ 1]; \; R = \text{diam}[10, 10]$$

were selected on the basis of low proportional differential and integral feedback gain values and such that the eigenvalues of the closed-loop airplane are similar to those of the closed-loop model. The gain values are

$$\left[\begin{array}{c|c} K_{PD_p} & K_{I_p} \\ \hline K_1 & K_2 \end{array}\right] = \left[\begin{array}{cccc|cc} -4.063 & -1.638 & 2.97 & -6.249 & -3.218 & 0.7598 \\ -1.989 & 4.331 & -4.298 & -1.791 & -0.796 & -3.176 \\ \hline 1.07 & 0.309 & -0.5032 & 1.403 & 0.36 & -0.313 \\ 0.451 & -0.848 & 1.94 & 0.658 & 0.0846 & 1.207 \end{array}\right]$$

Typical time history results are shown in Fig. 2. These results illustrate the decoupling and the zero state error properties and the good transient behavior for this formulation of the problem.

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Technical Comments

Comment on "A Finite-Element Method for Calculating Aerodynamic Coefficients of a Subsonic Airplane"

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WE wish to take issue with Hua[1] who contends that "In the steady aerodynamic forces area, Woodward,[2] Belotserkovskii,[3] Hedman,[4] and Kálmán et al.[5] have made their prominent contributions. However, most of their works are only concerning the longitudinal forces and moment. A general consideration about all six aerodynamic coefficients of an airplane has not been found in the literature." A general consideration of all six degrees of freedom *is* to be found in the literature if one recognizes that Refs. 2–5 have been concerned primarily with *distributions* of aerodynamic loads; the integration of these distributions to obtain six-degree-of-freedom aerodynamic coefficients is a trivial matter. Furthermore, Refs. 2–5 have been more fundamentally concerned with *aerodynamic influence coefficients* that are *independent* of any specific motion, so that calculation of load distributions arising from longitudinal, lateral, or directional motions is again trivial.¶

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¶The rotary cross-derivatives are, of course, exceptions to this, and so are other derivatives depending on second-order effects.[6-8]

We might also note that Hua's interest in performance is satisfied by a steady flow solution, but that his interest in flying qualities requires, in addition, an oscillatory solution.[9-11] However, that is another matter; it is our purpose here only to comment on Ref. 1, insofar as it is incorrect or inaccurate.

In the matter of wing-body interference, Refs. 2, 5, and 12 apply two different techniques to account for interference per se: Refs. 2 and 5 use lifting surface elements over the fuselage surface in the vicinity of the wing-fuselage juncture; Ref. 12 (with results also shown in Ref. 5) utilizes a wing image system, as well as an axially segmented line doublet within the fuselage, to simulate the interference. Both techniques approximate the fuselage as a cylinder. The approaches of Refs. 5 and 12 have been combined and extended in Ref. 11 to include unsteady wing-body effects. A slender body idealization of the fuselage is made to account for varying diameter, while the image system within a cylindrical interference surface is maintained to offset the effects of the lifting surface vortex (and doublet in the oscillatory case) system. The use of the image system for wing-body interference was first used by Multhopp[13] in 1941. In Ref. 1, however, we find no consideration of images, but find the assumption (presumption ?) that a source and doublet on the body axis will simulate fuselage interference and that the vortices on lifting panels give negligible contribution to the total force on a cone. Had Hua made a calculation of the spanwise loading on a midwing-fuselage combination, he would have found that his wing lift would have vanished at the side of the fuselage because of the lack of images. Also because the lifting surface induced lift is ignored, his fuselage lift will be considerably less than the lift that should carry across the fuselage. This defect is not so apparent in the example Hua has chosen, where the fuselage is tangent to the low wing at the centerline.

On the subject of drag, Hua notes that "the induced drag prediction might not be improved without modifying the method by including leading edge suction." Among the works "only concerning the longitudinal forces," we have contributed a Note[14] on the spanwise distribution of induced drag on lifting surfaces. It was based on a straightforward application of the Kutta-Joukowski Law, without any consideration of leading edge suction. Some controversy arose[7] over this application of the Kutta-Joukowski Law in general, and to estimate the lateral-di-